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Measuring Intertemporal Substitution: The Role of Durable Goods

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In estimating the intertemporal elasticity of substitution, Hall finds that, when one takes account of time aggregation, point estimates are small and not significantly different from zero. He concludes that the elasticity is unlikely to be much above 0.1 and may well be zero. Applying improved inference methods to an economic model similar to Hall's, Hansen and Singleton show that there is considerably less precision in the estimation. We argue that the model used by these authors is misspecified because the intratemporal substitution between nondurable consumption goods and durable consumption goods is ignored. We use a two-step procedure that combines a cointegration approach to preference parameter estimation with generalized method of moments to take these effects into account. Our estimates for the intertemporal elasticity

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of substitution are positive and significantly different from zero, even when time aggregation is taken into account.

I. Introduction

As Hall (1988) points out, intertemporal substitution by consumers is a central element of most modern macroeconomic models. The quantitative importance of effects of changes in various policies implied by these models depends on the magnitude of the intertemporal elasticity of substitution (IES) of consumption. In estimating the IES, however, Hall finds that his point estimates are small, sometimes negative, and not significantly different from zero when time aggregation is taken into account. Working with a similar economic model, Hansen and Singleton (1996) improved on Hall's inference methods with a technique that is scale invariant and asymptotically efficient. While they find that there is considerably less precision in the estimation and evidence against small positive values of the IES, their point estimates are negative.

Both Hall and Hansen and Singleton assume that preferences are additively separable in nondurable and durable goods, but there is empirical evidence against this assumption (see, e.g., Eichenbaum and Hansen 1990). In principle, when two goods are not additively separable, ignoring one good in estimating the IES of the other good does not necessarily induce a bias that increases the probability of finding either small and positive point estimates or estimates with the wrong sign. In the case of nondurable and durable goods, however, ignoring durable goods in estimating the IES, as in Hall and Hansen and Singleton, likely introduces a bias in this direction.

There are two reasons for this misspecification bias. First, real interest rates affect the user cost for the service flow from the purchase of the durable good. For example, suppose that the real interest rate rises this year in an economy in which the nondurable and durable goods are substitutes. Other things being equal, this results in a higher user cost for the durable good this year, and thus consumers will substitute away from the durable good and increase today's consumption of the nondurable good. As long as the user cost in the next year does not fall to offset this effect, the growth rate of nondurable consumption decreases compared with the case of no change in the user cost. Hence, the estimator of the IES that is based only on the growth rate in nondurable consumption will be biased downward. If the intratemporal substitution effect is strong enough, it can even cause the sign of the point estimate to be negative. In fact, we show that when we exclude durables by adopting the separability assumption in our data, we obtain small and negative point estimates of the IES. This finding is in contrast with our positive and significant point estimates of the IES when the separability assumption is relaxed.

Second, the service flow from the durable good service is likely to be more responsive to real interest rate changes than nondurable consumption. For example, Mankiw (1985) has estimated a larger IES for the consumption of durable goods than for nondurable consumption. Our approach differs from Mankiw's in that we mainly focus on the nonseparability of preferences in nondurable and durable goods whereas Mankiw assumes separability.

In order to see whether this misspecification bias is important, we use Cooley and Ogaki's (1996) cointegration–Euler equation approach and allow for nonseparable preferences in nondurable and durable goods. We assume that the constant elasticity of substitution (CES) utility function represents intratemporal preferences. The CES utility function is estimated by a cointegration regression in the first step. In the second step, generalized method of moments (GMM) is applied to the Euler equation with the estimated CES utility function. Cooley and Ogaki apply the same approach to estimate the IES for the nondurable good in a model that assumes separable preferences in consumption and leisure. Ostry and Reinhart (1992) follow a strategy similar to the one in this paper to estimate the preference parameters of a two-good model of traded and nontraded goods in a panel of developing countries. Ogaki, Ostry, and Reinhart (1996) extend this analysis by allowing the IES to vary with the level of wealth.

Dunn and Singleton (1986), Eichenbaum and Hansen (1990), and Fauvel and Samson (1991) estimate the parameters of Euler equations in models that allow for the nonseparability of preferences between nondurables and durables, though they do not focus on the bias in the estimates of the IES. The estimation methods in these papers do not allow for adjustment costs for durable goods. This is a serious problem because as Bernanke (1984), Lam (1989), and Eberly (1994), among others, have shown, adjustment costs are important determinants of durable good consumption. Our method is robust to various specifications of adjustment costs, relying on the cointegrating properties between the observed and the desired stock of durable goods in the presence of adjustment costs, which is discussed in Caballero (1993). In estimating the intratemporal elasticity in the first step, we use a cointegrating regression that utilizes long-run information to identify the elasticity. Hence, as long as adjustment costs do not affect the long-run behavior of durable good consumption, our estimator is consistent. In the second-step GMM estimation, we use the Euler equation obtained by considering changes in nondurable consumption, but not that for changes in durable consumption. It can be shown that the Euler equation for nondurable consumption is robust to various forms of adjustment costs for durable good consumption.

The rest of this paper is organized as follows. Section II presents our theoretical framework for nonseparable preferences in nondurable and durable consumption. Section III explains the data and reports summary statistics, and Section IV explains our econometric method. Section V contains the empirical results, and Section VI provides our concluding remarks.

II. Theoretical Framework

Suppose that a representative consumer maximizes the lifetime utility function

$$U = E_0\left[\sum_{t=0}^{\infty} \beta^t\left(\frac{\sigma}{\sigma-1}\right)[u(t)^{1-(1/\sigma)} - 1]\right], \quad \sigma > 0, \beta > 0, \quad (1)$$

in a complete market at time 0, where σ is the IES, β is the subjective discount factor, and $E_t[\cdot]$ denotes expectations conditional on the information available at time t. The intraperiod utility function is assumed to have the CES form for the nondurable good and the durable good:

$$u(t) = [aC(t)^{1-(1/\epsilon)} + S(t)^{1-(1/\epsilon)}]^{1/[1-(1/\epsilon)]}, \quad a > 0, \epsilon > 0, \quad (2)$$

where $C(t)$ is the consumption of the nondurable good, $S(t)$ is the service flow from the purchases of the durable good, ϵ is the intratemporal elasticity of substitution, and a is some positive number that determines the weight attached to the nondurable good in the intraperiod utility function. Purchases of the durable consumption good and the service flow are related by

$$S(t) = D(t) + \delta D(t-1) + \delta^2 D(t-2) + \cdots, \quad 0 < \delta < 1, \quad (3)$$

where $D(t)$ is the real consumption expenditure for the durable good at time t, and $1 - \delta$ is the depreciation rate of the durable good.

We take the nondurable good as a numeraire for each period and define $P(t)$ as the purchase price of the durable good in terms of the nondurable good. Let $R(t + 1)$ be the (gross) return on any asset in terms of the nondurable good that is realized at $t + 1$. Then the Euler equation is

$$E\left[\frac{\beta R(t+1)\mu(t+1)}{\mu(t)}\right] = 1, \tag{4}$$

where marginal intraperiod utility is defined as

$$\mu(t) = C(t)^{-1/\epsilon}[aC(t)^{1-(1/\epsilon)} + S(t)^{1-(1/\epsilon)}]^{(\sigma-\epsilon)/[\sigma(\epsilon-1)]}. \tag{5}$$

The user cost for the service flow of the durable good, $Q(t)$, is

$$Q(t) = P(t) - \delta E_t\left[\frac{\beta P(t+1)\mu(t+1)}{\mu(t)}\right]. \tag{6}$$

Because this formula involves the conditional expectation operator, it is complicated to calculate the user cost. For this reason, we shall derive a cointegration restriction that is based on the purchase price, $P(t)$, rather than on user cost. We shall then use the cointegration restriction to estimate the intraperiod elasticity, ϵ.

However, it is instructive to calculate a proxy for the user cost because the correlation between the user cost and the real interest rate is a factor that determines the misspecification bias in the single-good model discussed in the Introduction. When the correlation between the user cost and the real interest rate is positive, the single-good model estimator is biased toward zero and negative values. Hence, for the purpose of obtaining a proxy for the user cost, we adopt the approximation

$$Q(t) \cong P(t) - \frac{\delta E_t[P(t+1)]}{E_t[R(t+1)]}. \tag{7}$$

The following first-order condition states that the user cost is equal to the marginal rate of substitution of the service flow of the durable good and the nondurable good:

$$Q(t) = a^{-1}\left[\frac{S(t)}{C(t)}\right]^{-1/\epsilon}. \tag{8}$$

In order to derive the restrictions that imply cointegration, it is useful to observe another first-order condition that states that the purchase price of the durable good, $P(t)$, is equated with the marginal rate of substitution based on purchases of goods:

$$P(t) = \frac{\partial U/\partial C(t)}{\partial U/\partial D(t)} = \frac{E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \delta^\tau \mu_2(1+\tau)\right]}{\mu(t)}, \tag{9}$$

where

$$\mu_2(t) = S(t)^{-1/\epsilon}[aC(t)^{1-(1/\epsilon)} + S(t)^{1-(1/\epsilon)}]^{(\sigma-\epsilon)/[\sigma(\epsilon-1)]}. \tag{10}$$

This first-order condition forms the basis of the cointegration approach and summarizes the information from the demand side. In order to model the supply side in the simplest way, we consider an endowment economy without production. Let $C^*(t)$ and $D^*(t)$ be the endowments of the nondurable good and the durable good, respectively, and define $c^*(t) = \log[C^*(t)]$ and $d^*(t) = \log[D^*(t)]$. In equilibrium, then, $c(t) = \log[C(t)] = c^*(t)$, and $d(t) = \log[D(t)] = d^*(t)$.

In a production economy, we require that equilibrium consumption satisfies the assumed trend properties of $c^*(t)$ and $d^*(t)$. The trend properties of equilibrium consumption are likely to be closely related to those of the technology shock to the individual industry in a production economy.

Assume that $c^*(t)$ and $d^*(t)$ are difference stationary. Multiplying both sides of equation (9) by $[C(t)/D(t)]^{-1/\epsilon}$ yields

$$P(t)\left[\frac{C(t)}{D(t)}\right]^{-1/\epsilon} = E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \delta^\tau \left[\frac{S(t+\tau)}{D(t)}\right]^{-\epsilon} \left[\frac{C(t)}{C(t+\tau)}\right]^{-\epsilon} \frac{\mu(t+\tau)}{\mu(t)}\right], \tag{11}$$

which implies that the left-hand side is stationary if we assume that the discounted sum in the right-hand side can be forecasted optimally by the stationary variables in the information set. Hence the first-order condition, equation (9), implies that $P(t)[C(t)/D(t)]^{-1/\epsilon}$ is stationary under this assumption. The cointegration approach, which is used to estimate ϵ in step 1 of our two-step approach, is based on this stationarity restriction. In Section V, we test the empirical validity of these assumptions and restrictions.

We made an additional assumption that the discounted sum in the right-hand side of equation (11) can be forecasted optimally by the stationary variables in the information set. There may exist a problem with this additional assumption because it is stringent when the variables in the right-hand side of equation (11) are individually nonstationary. Even though $S(t+\tau)/D(t)$ and $C(t)/C(t+\tau)$ are stationary because $c^*(t)$ and $d^*(t)$ are difference stationary, $\mu(t + \tau)/\mu(t)$ is not strictly stationary. Hence, in principle, it is possible that this nonstationarity causes a problem. However, in practice, we do not expect this to be a serious problem because the nonstationarity of $\mu(t + \tau)/\mu(t)$ is unlikely to be empirically important, given that it is the growth rate of $\mu(t)$.[1]

Implicit in the cointegration approach, the stationarity restriction crucially requires that preferences are stable relative to the trends in equilibrium consumption expenditures. The most important factor that could cause problems with this assumption would probably be trending demographic changes. This, however, does not seem to cause practical problems for our data set. As Ogaki (1992) shows, preference parameters estimated by applying the cointegration approach to U.S. aggregate time-series data are consistent with some of the demand function properties that are estimated from cross-sectional household data while one controls for demographic factors.

III. Data and Summary Statistics

In this section, we describe the data set used in this paper and report selected summary statistics. The data are quarterly. For the nondurable good in the model, we use real expenditures on nondurable consumption minus clothing consumption. For the durable good in the model, we use real spending from Gordon's (1990) data. Gordon's data treat the quality improvement of durable goods in an arguably better way than the data from the National Income and Product Accounts (NIPA).[2] We use the implicit deflators as the purchase prices. Because Gordon's data are annual, we use the quarterly series that Ogaki and Park (1998) construct from Gordon's original data. In constructing the service flow series for durables, we used equation (3) with the initial condition on $S(t)$ from Musgrave (1979). In Musgrave's data, the depreciation rate is about 18 percent. Wykoff (1970) estimates a depreciation of about 20 percent per year using resale values of automobiles. For our base results, we use $\delta = 0.94$ for the quarterly data, which translates into an annual depreciation rate of about 22 percent. For a sensitivity analysis, we shall also use $\delta = 0.96$ and $\delta = 0.92$, which translate into annual depreciation

[1] In Ogaki and Reinhart (1997), we report that the null hypothesis of stationarity cannot be rejected for the growth rate of estimated $\mu(t)$.

[2] For results with nondurables plus services minus clothing and NIPA durable good quarterly data, see Ogaki and Reinhart (1997). For results with annual data for 1929–90, see Ogaki and Reinhart (1998).

TABLE 1

Summary Statistics

A

	Correlation Coefficient	Standard Error
User cost and real interest rate	.1460	.0563

B

	Standard Deviation	Standard Error
User cost	.0617	.0058

C

Growth Rates of Consumption and Expenditures	Standard Deviation	Standard Error
Durable good consumption	.0435	.0031
Durable good service flow	.0058	.0006
Nondurable good consumption	.0049	.0003

Note.—The standard errors are calculated by the COREST.EXP program in the Hansen/Heaton/Ogaki GAUSS GMM package (described in Ogaki 1993*b*, 1993*c*, 1998), using a VAR(1) prewhitened quadratic spectral kernel estimator with Andrews's (1991) automatic bandwidth selection. We use $\delta = 0.94$ to calculate the service flows and the user cost of durable goods.

rates of about 15 percent and 28 percent, respectively. In order to obtain per capita real consumption, we divided real consumption by total population including armed forces overseas (averaged over each quarter).

The 3-month Treasury bill rate measured at the end of each quarter and Barro and Sahasakul's (1983) average marginal tax rate series are used to construct nominal after-tax rates. These are converted into real rates by the implicit deflator for the nondurable good.

Because Gordon's data are available only up to 1983:4, the data cover only the period from 1947:1 to 1983:4. It should be noted that the stock of durable goods was very low immediately after World War II. This means that the stock of durable goods grew faster than the expenditures of durable goods in the period immediately following World War II. Because the cointegration regression depends on the assumption that the stock of durable goods grows at the same rate as the purchase of durable goods and because we want to exclude this unusual period of restocking, it is appropriate to start the sample period at a later date. For this reason, all the results pertain to the sample period starting 1951:1 unless otherwise noted.

Table 1 reports selected summary statistics for the data. Panel A of table 1 reports the correlation between the user cost and the real interest rate. We use a vector autoregression (VAR) with three lags for the realized real interest rate and the growth rate of the purchase price of the durable good to obtain the expected values of these variables for the calculation of the approximation of the user cost given by equation (7). We report the correlation of the growth rate of the user cost, $\ln(Q_t) - \ln(Q_{t-1})$, with the expected real interest rate. As discussed in the Introduction, when this correlation is positive, the estimator of the IES is likely to have the misspecification bias. As table 1 highlights, this correlation is estimated to be positive and significant at the 5 percent level.

Panel B of table 1 reports the standard deviation of the growth rate of the user cost of durable goods relative to the nondurable good price. The standard deviation is positive and statistically significant. Hence, there may be substantial bias in the estimation of the IES with the single-good model when it is applied to total consumption (calculated by adding up nondurable consumption and the service flow from durable good purchases). In other words, Hicks's aggregation is inappropriate when the relative price is not constant.

Panel C of table 1 reports the standard deviation of the growth rates of consumption and the service flow of durable good expenditures. The growth rate of each variable is calculated as the first difference of the log of the variable. We note that the growth rate of durable expenditures is much more volatile than that of nondurable consumption. The more relevant comparison for our purpose, however, is made between nondurable consumption and the service flow from durable good purchases. If the service flow of durable good expenditures was more volatile than nondurable consumption, that would be yet another element that causes a downward bias in the estimation of the IES for total consumption expenditure. In fact, the service flow is much smoother than the durable good purchases and is about as smooth as nondurable consumption. Even though the service flow of durable good expenditures is about as smooth as nondurable consumption, Mankiw's (1995) results imply that the service flow of durable good expenditures is more responsive to real interest rate changes than nondurable consumption.

Figure 1 plots $\ln[C(t)/D(t)]$ and its relative price, $\ln[P(t)]$. The sample period shown is 1951:1–1983:4. We draw attention to two features in the figure. First, nondurable consumption relative to durable expenditures (the dotted line) has tended to decline over time. Second, this trend is consistent with the downward drift in the relative price of durable goods (the solid line).

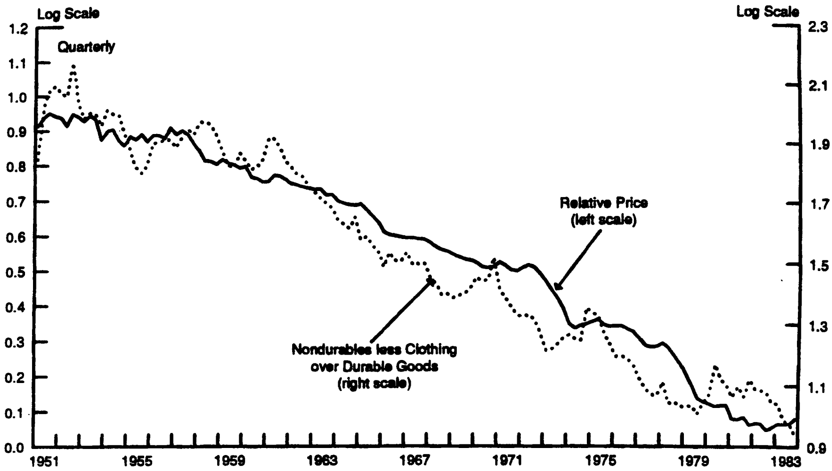

FIG. 1.—Durable expenditures (Gordon) relative to nondurables less clothing (NIPA).

IV. Estimation and Inference

In this section we describe our econometric method. We use Cooley and Ogaki's (1996) two-step procedure, which combines Ogaki and Park's (1998) cointegration approach to preference parameter estimation with Hansen and Singleton's (1982) GMM approach.

A. *Implications of the Intratemporal First-Order Condition*

The notions of stochastic and deterministic cointegration are useful when the economic variables of interest are modeled as difference stationary with drift. We focus here on processes that are integrated of order one. Suppose that the components of a vector series $\mathbf{X}(t)$ are difference stationary with drift. If a linear combination of $\mathbf{X}(t)$, $\gamma'\mathbf{X}(t)$, is trend stationary, the components of $\mathbf{X}(t)$ are said to be (stochastically) cointegrated, with a cointegrating vector γ. Consider the additional restriction that the cointegrating vector eliminates the deterministic trends as well as the stochastic trends, so that $\gamma'\mathbf{X}(t)$ is stationary. This restriction is called the deterministic cointegration restriction.

We assume that $c^*(t) - d^*(t)$ is difference stationary with drift. In Section II, we showed that the intratemporal first-order condition implies that $P(t)[C(t)/D(t)]^{-1/\epsilon}$ is stationary. This stationarity restriction implies that $p(t) = \ln[P(t)]$ is difference stationary, that $[p(t), c(t) - d(t)]$ is cointegrated with a cointegrating vector $(1, -1/\epsilon)$, and that the deterministic cointegration restriction is satisfied in equilibrium.

B. *Step 1: Cointegration*

This subsection describes our econometric procedure for the estimation of the cointegrating regression. This procedure allows us to test the null hypothesis of stochastic cointegration and the deterministic cointegration restriction.

Let $\mathbf{X}(t)$ be a two-dimensional difference stationary process: $\mathbf{X}(t) - \mathbf{X}(t-1) = \boldsymbol{\phi} + \mathbf{e}(t)$ for $t \geq 1$, where $\boldsymbol{\phi}$ is a two-dimensional vector of real numbers, $\mathbf{e}(t)$ is a stationary process with mean zero, and each component of $\mathbf{e}(t)$ has a positive long-run variance. Suppose that the $\mathbf{X}(t)$ are cointegrated, with a cointegrating vector $(1, -\gamma)$, and that the deterministic cointegration restriction is satisfied. Then we can apply Park's (1992) canonical cointegrating regressions (CCR) procedure[3] to

$$X_1(t) = \theta_c + \gamma X_2(t) + e_c(t). \tag{12}$$

In our empirical application, we can take either $p(t)$ or $c(t) - d(t)$ as a regressant. We use $c(t) - d(t)$ as a regressant, so that $\mathbf{X}(t) = [c(t) - d(t), p(t)]$, and the cointegrating vector is $(1, -\epsilon)$.

The CCR procedure requires us to transform the data before running a regression so as to correct for endogeneity and serial correlation. Let $\mathbf{v}(t) = (e_c(t), e_2(t))$, where $e_2(t)$ is the second element of $\mathbf{e}(t)$. Define $\boldsymbol{\Phi}(i) = E[\mathbf{v}(t)\mathbf{v}(t-i)']$, $\boldsymbol{\Sigma} = \boldsymbol{\Phi}(0)$, $\boldsymbol{\Gamma} = \sum_{i=0}^{\infty} \boldsymbol{\Phi}(i)$, and $\boldsymbol{\Omega} = \sum_{i=-\infty}^{\infty} \boldsymbol{\Phi}(i)$. Here $\boldsymbol{\Omega}$ is the long-run covariance matrix of $\mathbf{v}_t$. Define

$$\Omega_{11.2} = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21} \tag{13}$$

and $\boldsymbol{\Gamma}_2 = (\Gamma_{12}, \Gamma_{22})'$, where Ω_{ij} and Γ_{ij} are the ijth components of $\boldsymbol{\Omega}$ and $\boldsymbol{\Gamma}$, respectively. We make an additional assumption that $\Omega_{11.2}$ is positive. Consider transformations

$$X_1^*(t) = X_1(t) - \Pi_1'\mathbf{v}(t) \tag{14}$$

and

$$X_2^*(t) = X_2(t) - \Pi_2'\mathbf{v}(t). \tag{15}$$

[3] See Ogaki (1993*d*, 1998) for a more detailed explanation of CCR-based estimation and testing.

Because $\mathbf{v}(t)$ is stationary, $X_1^*(t)$ and $X_2^*(t)$ are cointegrated with the same cointegrating vector $(1, -\gamma)$ as $X_1(t)$ and $X_2(t)$ for any Π_1 and Π_2. The idea of the CCR is to choose Π_1 and Π_2 so that the ordinary least squares estimator is asymptotically efficient when $X_1^*(t)$ is regressed on $X_2^*(t)$. This requires

$$\Pi_1 = \Sigma^{-1}\mathbf{\Gamma}_2\gamma + (0, \Omega_{12}\Omega_{22}^{-1})' \tag{16}$$

and

$$\Pi_2 = \Sigma^{-1}\mathbf{\Gamma}_2. \tag{17}$$

In practice, long-run covariance parameters in these formulas are estimated, and the estimated Π_1 and Π_2 are used to transform $X_1(t)$ and $X_2(t)$. As long as these parameters are estimated consistently, the resultant CCR estimator is asymptotically efficient.

The CCR estimators have asymptotic distributions that can essentially be considered normal, implying that their standard errors have the usual interpretation.[4] An important property of the CCR procedure is that linear restrictions can be tested by χ^2 tests, which are free from nuisance parameters. We use χ^2 tests in a regression with spurious deterministic trends added to (12) in order to test for stochastic and deterministic cointegration. For this purpose, the CCR procedure is applied to the regression

$$X_1(t) = \theta_c + \sum_{i=1}^{q} \eta_i t^i + \gamma X_2(t) + e_c(t). \tag{18}$$

Let $H(p, q)$ denote the standard Wald statistic under the hypothesis $\eta_{p+1} = \eta_{p+2} = \ldots = \eta_q = 0$ with the estimate of the variance of $e_c(t)$ replaced by $\Omega_{11.2}$ (see Park [1990] for details). Then $H(p, q)$ converges in distribution to a χ^2_{q-p} random variable under the null of cointegration. In particular, the $H(0, 1)$ statistic tests the deterministic cointegrating restriction. On the other hand, the $H(1, q)$ statistic tests stochastic cointegration.

C. Step 2: The Estimation of the Intertemporal Elasticity of Substitution

In step 1, we obtain a consistent estimate of the intratemporal elasticity, ϵ. The second step of our procedure is to apply GMM to the

[4] The CCR estimators are asymptotically efficient, but there are other asymptotically efficient estimators such as those developed by Stock and Watson (1993), among others. Johansen's (1988) estimators are often used, but Johansen assumes a Gaussian VAR structure, which is not compatible with our economic model with nonlinear short-run dynamics.

Euler equation (4) in order to obtain estimates of intertemporal parameters. This two-step procedure does not alter the asymptotic distributions of the GMM estimators and test statistics because our cointegrating regression estimator is super-consistent and converges at a rate faster than $T^{1/2}$.

In step 1, we cannot obtain a consistent estimate for the preference parameter a. However, given an estimate of ϵ from step 1 and the approximation of $Q(t)$ explained in Section III, we can obtain a plausible value for a from equation (8), which, when terms are rearranged, yields

$$a = \exp\left\{\frac{\ln[C(t)/S(t)]}{\epsilon} - \ln[Q(t)]\right\}. \tag{19}$$

Using the approximated value of $Q(t)$, we obtain a plausible value of a by taking the exponential of the sample mean of $\{\ln[C(t)/S(t)]/\epsilon\} - \ln[Q(t)]$.

Given β, ϵ, and a, we apply GMM to the Euler equation (4) to obtain an estimate of σ. One possible problem here is the nonstationarity of $\mu(t+1)/\mu(t)$. As we discussed in Section II, the nonstationarity of this growth rate is unlikely to be empirically important, and hence we do not expect this to be a serious problem. Moreover, Gallant and White (1988) show that the stationarity assumption can be relaxed to obtain asymptotic results for GMM. Recent work by Andrews and McDermott (1995) and Dwyer (1995) shows that the stationarity assumption can be relaxed even further for asymptotic results.

In step 2 of GMM, the time aggregation problem is handled by lagging the instrumental variables two periods and by allowing the disturbance to have a moving average of order one (MA(1)) structure in the calculation of the optimum weighting matrix. Hall (1988) and Hansen and Singleton (1996) explain why this eliminates the time aggregation problem in some linear models. This does not completely remove the time aggregation problem in our nonlinear model. It should be noted that neither our method nor Hall's method (which is similar to ours) is perfect.[5] Even in Hall's linear model, we observe only the time average of the level of consumption rather than the time average of the log of consumption. For this

[5] Hall's econometric method assumes that the moving average coefficient for the disturbance is known, but we estimate the moving average coefficient in the GMM framework. We do not make the assumption that the moving average coefficient is known because the value of the coefficient can deviate from the value that Hall's theory predicts: e.g., the planning period of the consumer may be different from the one assumed by Hall.

TABLE 2

CANONICAL COINTEGRATING REGRESSION RESULTS

Sample Period	ϵ (1)	$H(0, 1)$ (2)	$H(1, 2)$ (3)	$H(1, 3)$ (4)	$H(1, 4)$ (5)
1947:2–1983:4	1.242	7.576	.004	.754	1.923
	(.098)	(.006)	(.947)	(.686)	(.589)
1951:1–1983:4	1.167	3.499	.009	1.750	2.499
	(.099)	(.062)	(.924)	(.417)	(.476)

NOTE.—Park and Ogaki's (1991) method with Andrews's (1991) automatic bandwidth parameter estimator was used to estimate long-run correlation parameters. In col. 1, standard errors are in parentheses. Col. 2 is a χ^2 test statistic for the deterministic cointegration restriction. Asymptotic P-values are in parentheses. Cols. 3–5 are χ^2 test statistics for stochastic cointegration. Asymptotic P-values are in parentheses.

reason, we try to avoid further time aggregation problems by using the point in time data of the interest rate rather than the time-averaged data.

V. Empirical Results

This section reports the results of the cointegrating regressions from step 1 for the intratemporal first-order condition and the step 2 GMM estimation of the Euler equation.[6]

Table 2 reports the cointegrating regression results from step 1 based on CCR.[7] When the full sample period of 1947:2–1983:4 is used, the $H(0, 1)$ test implies a strong rejection of the model. This result occurs probably because the level of the stock of durable goods was very low immediately after World War II. As discussed in Section III, this means that the stock of durable goods grew faster than the purchases of durable goods in the period immediately following World War II. Because the cointegration regression depends on the assumption that the stock of durable goods grows at the same

[6] In order to derive a cointegration relationship from the stationarity assumption, we made an additional assumption that $\ln[C(t)/D(t)]$ is difference stationary in Sec. IV. Unit root test results for this additional assumption are reported in Ogaki and Reinhart (1997).

[7] We used Ogaki's (1993*a*) GAUSS CCR Package for the CCR estimation. The CCR procedure requires an estimate of the long-run covariance of the disturbances in the system. We used Park and Ogaki's (1991) method with Andrews and Monahan's (1992) prewhitened heteroskedasticity and autocorrelation consistent estimator with the quadratic spectral kernel. A VAR of order one was used for prewhitening. We followed n. 4 of Andrews and Monahan, and the maximum absolute value of the elements of Δ notation was set to 0.99. Andrews's (1991) automatic bandwidth estimator, S_T, was constructed from fitting AR(1) to each disturbance. Following Monte Carlo–based recommendations by Park and Ogaki (1991), Han (1996), and Han and Ogaki (1997), we used the prewhitening method and report third-stage CCR estimates and fourth-stage CCR $H(p, q)$ test statistics.

TABLE 3

Generalized Method of Moments Results for the Two-Good Model

ϵ (1)	β (2)	a (3)	σ (4)	J_T (5)
1.167	.990	2.691	.447 (.072)	6.363 (.174)
1.167	.995	2.691	.329 (.049)	6.535 (.163)
1.167	.990	2.967	.438 (.073)	6.356 (.174)
.969	.990	2.691	.416 (.070)	6.384 (.172)

Note.—In col. 4, standard errors are in parentheses. Col. 5 reports Hansen's J test with four degrees of freedom; asymptotic P-values are in parentheses.

rate as the purchase of durable goods, it is appropriate to start the sample period at a later date in order to exclude this unusual period of restocking. For the sample period of 1951:1–1983:4, the $H(0, 1)$ test statistic is still significant at the 10 percent level but is not significant at the 5 percent level. None of the $H(1, q)$ test statistics is large enough to reject the model at the conventional significance levels.

For both sample periods, ϵ is estimated with the theoretically correct positive sign and is significantly different from zero. It is also estimated to be significantly greater than one at the 5 percent level for the full sample and significantly greater than one at the 10 percent level for the sample period of 1951:1–1983:4, so that the Cobb-Douglas utility function is rejected.

Tables 3 and 4 present the GMM results.[8] Table 3 presents the results for the two-good model described in Section II, and table 4 presents our results for the one-good model, which can be obtained by assuming $\sigma = \epsilon$ (i.e., separable preferences). For the one-good model, a is normalized to one. While the one-good model is similar to Hall's (1988) and Hansen and Singleton's (1996) model, we include the results because the estimation method and the sample period covered differ somewhat from Hall's. Unlike Hall, we do not linearize the Euler equation (4) because of the difficulty in doing

[8] We used the Hansen/Heaton/Ogaki GAUSS GMM package described in Ogaki (1993*b*, 1993*c*, 1998) that was supported by National Science Foundation grants SES-3512371 and SES-9213930 for the GMM estimation. We iterated on the weighting matrix as described by Kocherlakota (1990) up to four iterations, because his Monte Carlo results indicated that the iteration improves the small-sample properties of the GMM estimator.

TABLE 4

GENERALIZED METHOD OF MOMENTS RESULTS FOR THE ONE-GOOD MODEL

β (1)	σ (3)	J_T (3)
.990	−.186 (.074)	1.415 (.702)

NOTE.—In col. 2, standard errors are in parentheses. Col. 3 reports Hansen's *J* test with three degrees of freedom; asymptotic *P*-values are in parentheses.

so for the two-good model. We use exactly the same econometric method and data for both the one-good and two-good cases, so that we can directly compare the results.

The instrumental variables are a constant, the realized real interest rate, the growth rate of $C(t)$, the growth rate of $C(t)/D(t)$, and the growth rate of real defense expenditures. All instruments are lagged two periods rather than one. Since we were not able to obtain convergence when β is estimated with σ,[9] we report results when β is fixed. It is also convenient to fix β for the purpose of comparing the two-good and the single-good models. It is well known that β and σ are negatively related when Euler equations are estimated for the U.S. data. Hence it is difficult to compare estimates of σ for the two models when β is not fixed.

The first row of table 3 reports the base run results for the two-good model when β is fixed at 0.990. In an economy without growth, $\beta = 0.990$ implies a real interest rate of about 4.1 percent. For each of the base runs, the value of ϵ is fixed at the point estimate of ϵ from table 2 for the sample period that begins in 1951:1; the value of a is fixed at a value obtained by the method described in Section IV.

For the base run, Hansen's *J* test does not reject the model at the conventional significance levels. As column 4 of table 3 shows, our point estimates of σ are positive and significantly different from zero at the conventional significance levels.

To examine the robustness of our results, we conducted a series

[9] For the GMM results, we penalize the exceptionally high values of σ. For this purpose, we multiply the disturbance term by $1 + (|\sigma| - 10)^2$ when the absolute value of σ used in the nonlinear search program is greater than 10. When β is estimated, we penalize exceptionally low values of β. We multiply the disturbance term by $1 + 100{,}000(|\beta| - 0.98)$ when β is smaller than 0.98. The disturbance terms are not multiplied by these functions unless the bounds are reached. These bounds were sometimes reached in earlier iterations for the weighting matrix but were never reached in the last iteration reported in the tables.

of alternative runs in which the values of β, a, and ϵ were allowed to vary over a range of plausible values. The second to fourth rows of table 3 summarize the results of our sensitivity analysis with respect to the preset preference parameters.[10] The second row of table 3 reports sensitivity analysis results with respect to the choice of β; the third row, with respect to a; and the fourth row, with respect to ϵ. We change one parameter at a time, while keeping the values of the other two parameters fixed at the same values as those in the base runs, and check how the estimate of σ varies and if it is still significantly different from zero.

The second row of table 3 reports the results when the value of the discount factor, β, is increased to 0.995. In an economy without growth, $\beta = 0.995$ implies a real interest rate of about 2 percent. Because estimates of σ and β are negatively related for the U.S. data, we expect the point estimate of σ with $\beta = 0.995$ to be lower than that in the corresponding base run. The point estimate of σ is lower as expected, but it is still positive and significantly different from zero at the conventional significance levels. The J-test statistics are not very sensitive to changes in the value of β.

The third row of table 3 reports the results when the value of a is increased. The durable good is less important than the nondurable good in the representative consumer's preferences when the value of a is higher. Therefore, as long as the role of durable goods is important in obtaining higher point estimates of σ, the point estimates of σ are expected to be lower when the value of a is increased. In order to find a reasonable range for this parameter, we compute the standard error for a by treating the method of obtaining a plausible value of a as an estimation exercise given data for $C(t)$, $S(t)$, $Q(t)$, and ϵ. We apply the GMM, using a VAR(1) prewhitened quadratic spectral kernel estimator. The standard error is 0.138. We report results when a is raised by two standard errors. The point estimate of σ changes very little, though it is lower as expected; the J-test statistics are also not sensitive to changes in the value of a.

The fourth row of table 3 reports the sensitivity analysis results for ϵ. Even though the cointegrating regression estimator in step 1 is super-consistent and the asymptotic distributions of the step 2 estimators are not affected by step 1 estimation, their small-sample dis-

[10] Results for $\delta = 0.96$ and $\delta = 0.92$ are similar to the base run results. For example, σ is estimated to be 0.346 with the standard error of 0.058 when $\delta = 0.96$, and σ is estimated to be 0.320 with the standard error of 0.056 when $\delta = 0.92$ for nondurables and the NIPA durables. We also tried a different set of the instrumental variables for nondurables and NIPA durables without subtracting clothing consumption. The results were similar to those reported here and are available from the authors on request.

tributions are affected. Therefore, it is desirable that the point estimates, standard errors, and the *J*-test statistics are not sensitive with respect to the choice of ϵ. There is no role for durable goods in Euler equation (4) if $\epsilon = \sigma$, and durable goods become less important as ϵ approaches σ. Therefore, as long as the role of durable goods in Euler equation (4) is important in obtaining higher point estimates of σ, the point estimates of σ are expected to be lower when the value of ϵ is decreased. We report results when ϵ is decreased by two standard errors in table 3. The point estimate of σ changes very little, though it moves in the expected direction. As before, the *J*-test statistics are not sensitive to changes in the value of ϵ. The results that the point estimates of σ move in the expected direction in the sensitivity analyses for a and ϵ provide further support for our view that the role of durable goods is important in correctly measuring intertemporal substitution.

In all cases in table 3, the separability assumption ($\sigma = \epsilon$) is rejected. The point estimate of the IES, σ, in step 2 of GMM is smaller than that of the intratemporal elasticity of substitution, ϵ, in step 1 of the cointegrating regression by more than two standard errors in each case.

In table 3, when preferences are allowed to be nonseparable, the estimates of the IES fall in a relatively narrow range of 0.32–0.45. In contrast, the one-good model with the separability assumption (table 4) yields negative point estimates of σ.

VI. Conclusions

In this paper, we have argued that ignoring the intratemporal substitution between nondurables and durables is likely to result in a misspecification bias that increases the probability of finding small positive point estimates or even negative point estimates of the IES. When we account for this intratemporal substitution, our empirical results are very different from those of Hall (1988) and Hansen and Singleton (1996). The IES is estimated to be positive and significant, and the point estimates of the IES under the nonseparability assumption range from 0.32 to 0.45. Our result that the point estimate of the IES is positive and significant is quite robust to the assumptions made about the other preference parameters. In contrast, the point estimates of the IES based on our one-good model under the separability assumption are negative.

We have found empirical evidence against separability of preferences between nondurable and durable goods.[11] In particular, our

[11] Viard (1997) finds some evidence that the nonseparability may explain the predictability of nondurable consumption growth.

empirical results indicate that the intratemporal elasticity between nondurable and durable goods is much higher than the IES. This finding, together with the fact that part of durable good purchases are theoretically one type of saving, suggests that some of the puzzling behavior we observe with regard to saving may be explained by the addition of the intratemporal substitution between nondurable and durable consumption goods to standard models of saving.

In this paper, we have assumed that preferences are homothetic over nondurable and durable goods. This assumption may be counterfactual and may affect our estimates of the intratemporal elasticity of substitution in cointegrating regressions. One way to allow for nonhomotheticity is to allow for fixed subsistence levels. Because Atkeson and Ogaki (1996) found little effect of including fixed subsistence levels on regression coefficients for their U.S. data in a cointegration model, we do not expect that it would substantially affect our results. However, it is still of interest to consider other forms of nonhomotheticity together with nonseparability for nondurable and durable goods in preferences. This is an agenda for our future research.

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